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# Implementation of adiabatic geometric gates with superconducting phase qubits

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**Abstract**
We present an adiabatic geometric quantum computation strategy based on the non-degenerate energy eigenstates in (but not limited to) superconducting phase qubit systems. The fidelity of the designed quantum gate was evaluated in the presence of simulated thermal fluctuations in a superconducting phase qubits circuit and was found to be quite robust against random errors. In addition, it was elucidated that the Berry phase in the designed adiabatic evolution may be detected directly via the quantum state tomography developed for superconducting qubits. We also analyze the effects of control parameter fluctuations on the experimental detection of the Berry phase.

(Some figures in this article are in colour only in the electronic version)

## 1. Introduction

Quantum computation (QC) has been attracting more and more interest over the past decade due to its unrivaled power, which exceeds that of its classical counterpart in solving certain problems [1]. From the point of view of scalability, the solid-state system is regarded as indispensable for developing practical quantum computers. It has been realized that superconducting qubits provide us with a promising approach towards a scalable solid-state quantum computer [2–6]. Two-qubit gates have also been demonstrated in superconducting qubits [7–11]. However, it still necessary to realize gate errors small enough to achieve a fault tolerance threshold which is in the $10^{-4}$ range [12]. The fidelities of quantum gates in physical qubits based on microscopic particles, such as trapped ions [13], nuclear magnetic resonance [14], have achieved values of the order of 0.98. However, the unavoidable coupling with the environment in solid-state qubits results in a serious reduction of the fidelities of the desired quantum gates. Recently, achieving fidelities of quantum gates has been reported in superconducting qubits experiment with $F \sim 0.4$ [10]. Constructing fault-tolerant quantum logic gates in superconducting qubits based on the geometric phase [15] has been paid particular attention recently [16–19]. All of these efforts mainly focused how to implement universal quantum gates based on the geometric phases. To the best of our knowledge, the analysis of geometric quantum gate precision in a noisy environment is still awaited. Being different from the above schemes, we here develop an adiabatic geometric QC scheme based on the non-degenerate energy eigenstates. We address the realization of two-qubit gates when the qubit–qubit interaction is $\sigma_y\sigma_y$ type, such as in superconducting phase qubits. We mainly analyze the fidelities of adiabatic geometric quantum gates against certain kinds of simulated noises in superconducting phase qubits in real experimental conditions. Fianlly, a scheme based on state tomography is proposed to detect the Berry phase in superconducting phase qubits, which is different to the interference measurement scheme in [16].

The paper is organized as follows. In section 2, we introduce the adiabatic geometric gates. In section 3, we simulate the fidelities of both the single-qubit and the two-qubit controlled adiabatic geometric gate in the presence of random fluctuations. In section 4, the detection of the Berry phase in superconducting phase qubits is analyzed. Section 5 presents relevant discussions and a brief summary.

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## 2. Adiabatic geometric gates

We now elaborate our adiabatic geometric QC strategy. A qubit system, where the system Hamiltonian with two instantaneous non-degenerate energy levels changes adiabatically and cyclically in a parameter space with period $\tau$, behaves like a spin $\frac{1}{2}$ particle in a magnetic field, with the Hamiltonian $H = \vec{\sigma} \cdot \mathbf{B}/2$. Under the adiabatic approximation, the two orthogonal energy eigenstates $|\psi_\pm(t)\rangle$ will also follow the Hamiltonian to evolve adiabatically and cyclically starting from the initial states $|\psi_\pm(0)\rangle$: $|\psi_\pm(\tau)\rangle = U(\tau)|\psi_\pm(0)\rangle \approx \exp(\pm i\gamma)|\psi_\pm(0)\rangle$, where the $U(\tau)$ is the evolution operator of the system and the $\pm\gamma$ are respectively the total phases accumulated for the $|\psi_\pm\rangle$ states in the evolution. We denote $|\psi_+(0)\rangle = \cos\frac{\xi}{2}|0\rangle + e^{i\eta}\sin\frac{\xi}{2}|1\rangle$ and $|\psi_-(0)\rangle = -\sin\frac{\xi}{2}|0\rangle + e^{i\eta}\cos\frac{\xi}{2}|1\rangle$, with $|0\rangle$ and $|1\rangle$ as the two eigenstates of $\sigma_z$ chosen as our computational basis ($\eta = 0$ for $B_y(0) = 0$ and $\eta = \pi/2$ for $B_x(0) = 0$).

Thus, for an arbitrary initial state of the system $|\psi_{\rm in}\rangle = a_+|\psi_+(0)\rangle + a_-|\psi_-(0)\rangle$ with $a_\pm = \langle\psi_\pm(0)|\psi_{\rm in}\rangle$, after the adiabatic and cyclic evolution time $\tau$, the final state is found to be $|\psi_{\rm f}\rangle \approx U(\gamma, \xi, \eta)|\psi_{\rm in}\rangle$, where

$$U = \begin{pmatrix} e^{i\gamma}\cos^2\frac{\xi}{2} + e^{-i\gamma}\sin^2\frac{\xi}{2} & ie^{-i\eta}\sin\xi\sin\gamma \\ ie^{i\eta}\sin\xi\sin\gamma & e^{i\gamma}\sin^2\frac{\xi}{2} + e^{-i\gamma}\cos^2\frac{\xi}{2} \end{pmatrix}. \tag{1}$$

Moreover, a controlled two-qubit gate may also be achieved under the condition that the control qubit is off resonance in the operation of the target qubit (to be addressed later).

Considering that $\gamma$ is the total phase, usually consisting of both geometric and dynamic phases, we here illustrate how to eliminate the corresponding dynamic phase in a simple two-loop quantum gate operation, so that the achieved $U$-gate is a pure geometric one depending only on the geometric phase accumulated in the whole evolution. We set the basic adiabatically cyclic evolution time to be $\tau_0$ with the corresponding geometric Berry phase as $\gamma_g^0$. After the first cyclic evolution of the states $|\psi_\pm\rangle$ by driving the fictitious field adiabatically with period $\tau_0$, we promptly reverse the fictitious field direction such that the states $|\psi_\pm\rangle$ are unchanged, i.e. $\mathbf{B}(\tau_0+0) = -\mathbf{B}(\tau_0)$ and $|\psi_\pm(\tau_0+0)\rangle = |\psi_\pm(\tau_0)\rangle$. Then we let $\mathbf{B}(\tau_0 + t) = -\mathbf{B}(t)$ in the second $\tau_0$-time cycle evolution. During the second period, the state $|\psi_+\rangle$ ($|\psi_-\rangle$) acquires the same geometric phase as that in the first period but with the reversal sign of the dynamic phase, so that the accumulated total phase of $|\psi_+\rangle$ ($|\psi_-\rangle$) at the end of the second period is a pure geometric phase $\gamma = 2\gamma_g^0$ ($-2\gamma_g^0$). Therefore, the pure geometric quantum $U(2\tau_0)$-gates given by equation (1) can be obtained. For example, two simple non-commutable single-qubit gates, a type of Hadamard gate and a type of NOT gate, can be achieved by setting ($\xi = \pi/4$, $\eta = 0$, $\gamma_g^0 = \pi/4$) and ($\xi = \pi/2$, $\eta = 0$, $\gamma_g^0 = \pi/4$), respectively.

## 3. Fidelity of adiabatic gates

Recently, it was reported from numerical simulations that the earlier proposed two kinds of geometric quantum gates, a class of non-Abenian holonomic gates [20] and a set of nonadiabatic geometric gates [19, 21], are likely to be more robust against stochastic control errors than dynamical gates [22, 23]. It is natural to ask whether the present adiabatic geometric gates are also robust against stochastic errors as expected. To answer this question, here we illustrate this by superconducting phase qubits.

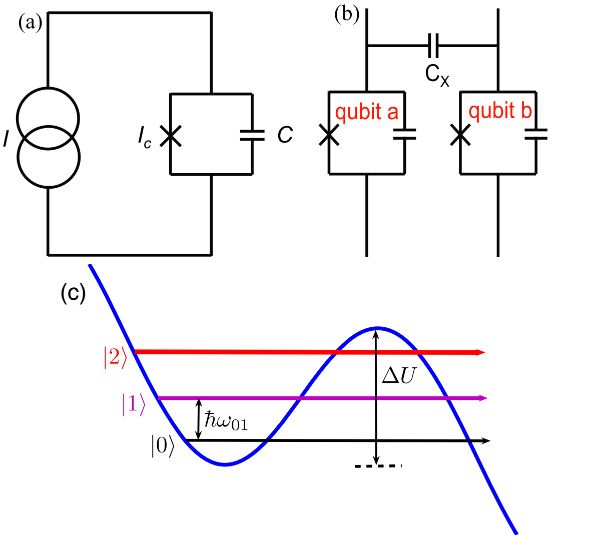


**Figure 1.** Schematic diagrams of (a) a circuit of a phase qubit; (b) a two-qubit gate, where two single phase qubits are coupled by a capacitor $C_x$; (c) quantized energy levels in a current-biased Josephson Junction, where the two lowest eigenstates $|0\rangle$ and $|1\rangle$ form a qubit.

As is known, a large current-biased Josephson junction (figures 1(a) and (c)) may work as a typical phase qubit, which can be considered as an anharmonic $LC$ resonator with resonance frequency $\omega_p = (L_J C_J)^{-1/2}$, whose two lowest quantized energy levels are chosen as the qubit states [5, 6, 24], where $L_J$ is the Josephson inductance and $C_J$ is the junction capacitance. The Josephson inductance is given by $L_J = \phi_0/2\pi I_c\cos\delta$, where $I_c$ is the junction critical current, $\delta$ is the phase difference across the junction given through $I = I_c\sin\delta$, and $\phi_0 = h/2e$ is the superconducting flux quantum. As the junction bias current $I$ becomes close to the critical current $I_c$, the anharmonic potential may be approximated by a cubic potential parameterized by the potential barrier height $\Delta U(I) = (2\sqrt{2}I_c\phi_0/3\pi)[1 - I/I_c]^{3/2}$ and a plasma oscillation frequency at the bottom of the well $\omega_p(I) = 2^{1/4}(2\pi I_c/\phi_0 C)^{1/2}[1 - I/I_0]^{1/4}$. Microwaves induce transitions between levels at a frequency $\omega_{mn} = E_{mn}/\hbar = (E_m - E_n)/\hbar$, where $E_n$ is the energy of state $|n\rangle$. The state of the qubit can be controlled with dc and microwave pulses of bias current $I(t) = I_{\rm dc} + \delta I_{\rm dc}(t) + I_{\mu w}(t)\cos\phi\cos\omega_{10}t + I_{\mu w}(t)\sin\phi\sin\omega_{10}t$. As usual, under a reasonable approximation that the dynamics of the system is restricted to the Hilbert space spanned by the lowest two states, the Hamiltonian in the $\omega_{10}$ rotating frame may be written as

$$\begin{aligned} H = {} & \hat{\sigma}_x I_{\mu w}(t)\cos\phi\sqrt{\hbar/2\omega_{10}C}/2 \\ & + \hat{\sigma}_y I_{\mu w}(t)\sin\phi\sqrt{\hbar/2\omega_{10}C}/2 \\ & + \hat{\sigma}_z\delta I_{\rm dc}(t)(\partial E_{10}/\partial I_{\rm dc})/2, \end{aligned} \tag{2}$$

where $\hat{\sigma}_{x,y,z}$ are Pauli operators. As schematically shown in figure 1(b), a non-trivial two-qubit gate could be constructed by capacitive coupling.

From equation (2), one could define a fictitious field $\mathbf{B} \equiv (\nu\cos\phi, \nu\sin\phi, \Delta\omega)$, where $\nu = I_{\mu w}(t)\sqrt{\hbar/2\omega_{10}C}$, $\Delta\omega = \delta I_{\mathrm{dc}}(t)(\partial E_{10}/\partial I_{\mathrm{dc}})$. The phase qubit thus behaves like a spin-$\frac{1}{2}$ particle in a magnetic field, with the Hamiltonian $H = \vec{\sigma}\cdot\mathbf{B}/2$. For such a quantum system, the acquired geometric phase of its energy eigenstate is equal to half of the solid angle subtended by the area in the parameter space enclosed by the closed evolution loop of the fictitious magnetic field. The solid angle may be evaluated by [19]

$$\Omega = \int_0^\tau \frac{B_x\partial_t B_y - B_y\partial_t B_x}{|B|(B_z + |B|)}\,\mathrm{d}t, \qquad (3)$$

under the condition $\mathbf{B}(\tau) = \mathbf{B}(0)$. In particular, when the adiabatic evolution path forms a cone in the parameter space $\{\mathbf{B}\}$ under varying current, the corresponding Berry phases of two energy eigenstates are simply given by [15] $\gamma_{\mathrm{g}} = \pm\pi[1 - \Delta\omega/\sqrt{(\Delta\omega)^2 + (\nu)^2}]$.

We perform numerical simulations on the fidelity of the adiabatic Berry phase gates given by equations (1) and (8), subject to the modeled random noises for the weakly fluctuated driving bias current. Note that in the numerical studies of [22, 23] the fluctuations of control parameters were assumed to be uniformly distributed in an interval and only certain types of states in the Bloch sphere were sampled to evaluate the average fidelity of gates. In real experimental conditions, the finite impedance of the bias-current source produces decoherence of the superconducting phase qubit from the dissipation and noises. We here consider the noise in the current due to the thermal fluctuation, which is probably one of the main noise sources in the superconducting qubits circuit [25]. The actual noise current generated by a resistance $R$ at temperature $T$ may be estimated by $I_n(\mathrm{rms}) = (4k_{\mathrm{B}}TB/R)^{1/2}$, where $B$ is the bandwidth parameter [26]. It is a white noise and its amplitude would obey a Gaussian distribution. In fact, assuming the critical current of superconducting phase qubits $I_{\mathrm{c}} \sim 10\ \mu\mathrm{A}$ [27], the measurement bandwidth $B \sim 10$ GHz, and the bias resistor $R \sim 10\ \mathrm{K}\Omega$ placed at the 4 K flange stage $T \sim 4.2$ K, the total current noise would be around 15 nA. The bias current is driven close to the critical current $I_{\mathrm{c}}$ and the transition frequency between qubit states is $\omega_{10}/2\pi \sim 6$ GHz. The Rabi frequency is $\nu/2\pi \sim 300$ MHz and the Ramsey frequency is $\Delta\omega/2\pi \sim 300$ MHz. The fluctuation of the Ramsey frequency resulting from noise in the bias current is about 10 MHz. Below we will evaluate the average fidelity of the designed new geometric quantum gates subject to this type of error for any input state.

As is known, the average fidelity of a quantum logic gate in the presence of random noises may be defined as

$$\overline{F} = \lim_{N\to\infty}\frac{1}{N}\sum_{j=1}^{N}|\langle\psi_{\mathrm{in}}|\widehat{U}^{\dagger}\widehat{U}^{j}_{\mathrm{noise}}|\psi_{\mathrm{in}}\rangle|^2, \qquad (4)$$

where $|\psi_{\mathrm{in}}\rangle = [\cos(\theta/2), \mathrm{e}^{\mathrm{i}\varphi}\sin(\theta/2)]^{\mathrm{T}}$ (T represents the transposition of matrix.), $\theta \in [0, \pi]$ and $\varphi \in [0, 2\pi]$ are the coordinators of the input state in our numerical simulations. Here, $U$ is an ideal adiabatic quantum gate denoted by equation (1) in the absence of random errors and $U_{\mathrm{noise}}$ is the gate operator in the presence of random errors.

For simplicity, we here focus only on a cone-type adiabatic evolution: $\xi = \tan^{-1}(\nu/\Delta\omega)$. Since this type of adiabatic evolution of the field could have various forms, it seems extremely difficult to directly simulate the state evolution in a reliable way under the adiabatic condition in the presence of random errors. To evade this difficulty, we adopt a simple method to model effectively the effect of random errors occurring in the evolution. For a given configuration of errors in the evolution, let us look at the final state $|\tilde{\psi}_+\rangle = U_{\mathrm{noise}}|\psi_+\rangle(0)$ and regard it to be evolved adiabatically and cyclically as well as ideally from a visual initial state $|\tilde{\psi}_+\rangle(0) = \cos\frac{\tilde{\xi}}{2}|0\rangle + \mathrm{e}^{\mathrm{i}\tilde{\eta}}\sin\frac{\tilde{\xi}}{2}|1\rangle = U^{-1}(\tilde{\gamma}, \tilde{\xi}, \tilde{\eta})|\tilde{\psi}_+\rangle$, namely, $|\tilde{\psi}_+\rangle = \mathrm{e}^{\mathrm{i}\tilde{\gamma}}|\tilde{\psi}_+\rangle(0)$. In this sense, $U_{\mathrm{noise}}$ evolved from the initial state may be expressed as $U(\tilde{\gamma}, \tilde{\xi}, \tilde{\eta})$ in equation (1) evolved from the corresponding visual initial state with $\tilde{\gamma}$ as the geometric Berry phase of the above mentioned two-loop evolution. Here, the random parameters $(\tilde{\gamma}, \tilde{\xi})$ may be determined by the randomly fluctuating bias current from the relations $\gamma_{\mathrm{g}}^0$ $(\nu, \Delta\omega)$ and $\xi$ $(\nu, \Delta\omega)$, with the Gaussian-type error probability density

$$\frac{\mathrm{d}p(x)}{\mathrm{d}x} = \frac{\exp(-x^2/2\sigma^2)}{\sqrt{2\pi}\sigma}, \qquad (5)$$

where the $x$ is the deviation from $\nu$ (or $\Delta\omega$, $\eta$), and $\sigma$ is the mean squared noise.

### *3.1. Fidelity of single-qubit gates*

In the numerical simulations reported here, we randomly choose more than ten thousand stochastic numbers ($N \geqslant 10\,000$) for a given mean squared noise $\sigma$ (for brevity but without loss of generality, we hereafter set $\tilde{\eta} = \eta = 0$ and neglect its randomness). The extracted random numbers during the evolution consider the ratio between the noise correlation time and the required time for adiabatic evolution (to be addressed later). We select the experimental parameter of superconducting phase qubits: $\omega_{10}/2\pi = 6$ GHz and $\sigma_0 = \sigma_1 = 0.1$, where $\sigma_0$ and $\sigma_1$ represent the fluctuation of $\Delta\omega$ and $\nu$ respectively. We then calculate the average fidelity (up to satisfactory convergence) versus the coordinates of the initial state and the parameters, as depicted in figures 2, 3, and 4, respectively. Several remarkable features can be seen from the figures. (i) The calculated fidelity of geometric quantum gates for any input state is rather high (larger than 0.92) for the considered noises. Actually, the amplitude of the microwave current could be controlled precisely in experiments. Therefore, the $\sigma_1$ is much smaller than $\sigma_0$ in superconducting phase qubits. From the above discussions, if the $\sigma_0$ is about 0.03, the designed geometric quantum gate is most likely to be rather insensitive to the stochastic errors. (ii) The suppression effect of $\Delta\omega$ fluctuations on the fidelity is weaker than that of $\nu$ fluctuations, and this becomes more pronounced when the mean squared noise is stronger (not shown here). Also reasonably, the joint effect of both $\Delta\omega$ and $\nu$

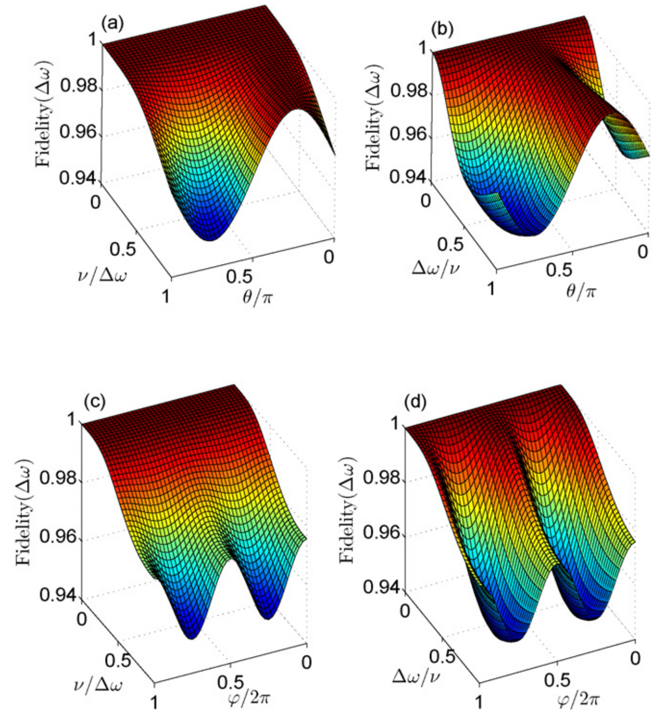


**Figure 2.** The fidelity of the single-qubit gate in the presence of $\Delta\omega$-fluctuations, where $\varphi = 0$ in (a) and (b), $\theta = \pi/2$ in (c) and (d). Parameters are: $\omega_{10}/2\pi = 6$ GHz and $\sigma_0 = 0.1$.

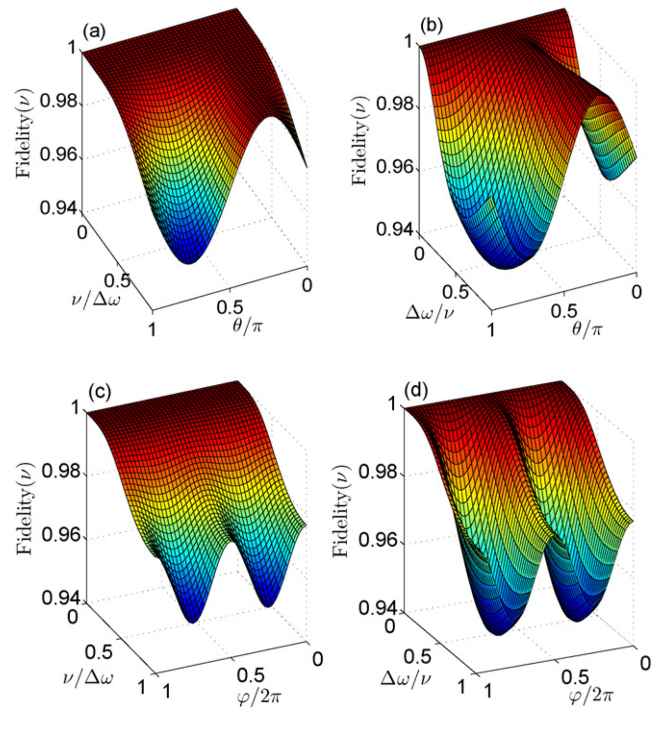


**Figure 3.** The fidelity of a single-qubit gate in the presence of $\nu$-fluctuations, where $\varphi = 0$ in (a) and (b), $\theta = \pi/2$ in (c) and (d). Parameters are: $\omega_{10}/2\pi = 6$ GHz and $\sigma_1 = 0.1$.

fluctuations on the fidelity is stronger than any single one, but the shapes of the corresponding figures are similar. Note that since it seems difficult to control the $\Delta\omega$ precisely because $\Delta\omega$

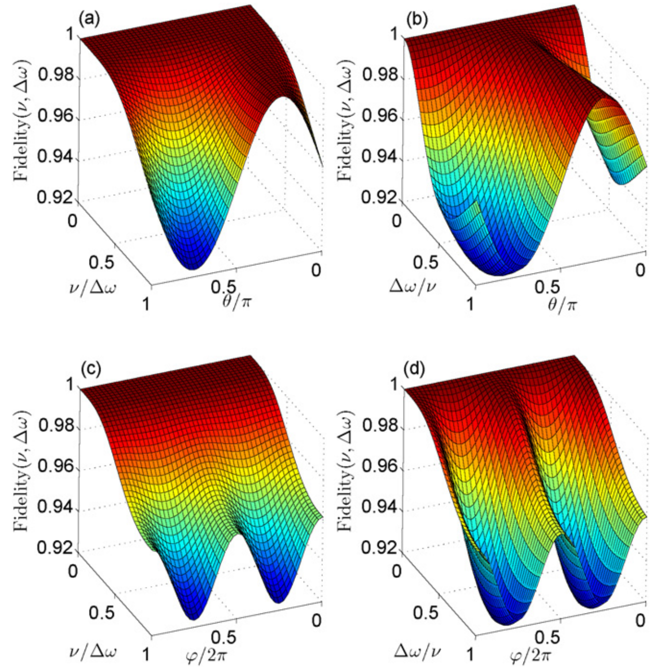


**Figure 4.** The fidelity of a single-qubit gate in the presence of fluctuations on both $\Delta\omega$ and $\nu$, where $\varphi = 0$ in (a) and (b), $\theta = \pi/2$ in (c) and (d). Parameters are: $\omega_{10}/2\pi = 6$ GHz, $\sigma_0 = 0.1$, and $\sigma_1 = 0.1$.

often varies due to the noise current in experiments [28], to optimize a quantum gate with the present geometric scenario may be quite helpful. (iii) The fidelity is very close to 1 for $\nu \ll \Delta\omega$ or $\nu \gg \Delta\omega$. Actually, we have a trivial geometric phase $2\pi$ and a trivial unit gate in this case. (iv) For a given $\xi$, the fidelity reaches a maximum when the input state is the eigenstate of the Hamiltonian.

### *3.2. Fidelity of two-qubit gates*

We now turn to address a kind of nontrivial two-qubit controlled phase gate in the present system. So far, a number of research efforts have been made about the two-qubit controlled phase gate [16, 17, 19, 29, 30]. Here we consider a system consisting of two superconducting phase qubits coupled with a capacitor, as shown in figure 1(b), whose Hamiltonian may be given by

$$\hat{H} = \sum_{i=a,b} \hat{H}_i + \frac{J}{2}\hat{\sigma}_y^{(a)}\hat{\sigma}_y^{(b)}, \tag{6}$$

where the coupling strength $J \approx (C_x/C_J)\hbar\omega_{01}$. This Hamiltonian could be used to manipulate the target qubit (*qubit b*) for the realization of a two-qubit gate under the condition that the control qubit (*qubit a*) is off resonance in the operation of the target qubit. The Hamiltonian of *qubit b* in the $\omega_{10}^b$ rotating frame is dependent on the state of *qubit b* through the coupling term $J$: the contribution is $J/2$ (or $-J/2$) if the state of *qubit a* is $|\psi_a\rangle = |-\rangle$ (or $|\psi_a\rangle = |+\rangle$), with $|-\rangle$ and $|+\rangle$ as the two eigenstates of $\sigma_y$. Setting $\tilde{\eta} = \eta = \pi/2$ and after an adiabatic evolution loop, the acquired geometric phase of the

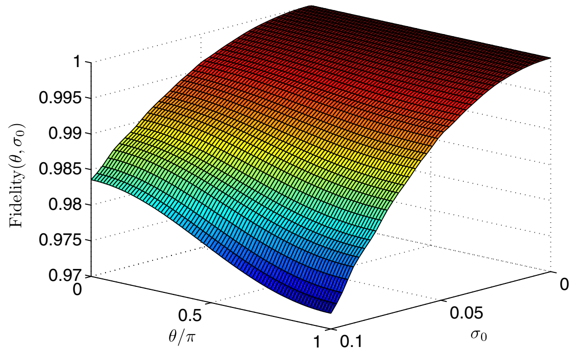


**Figure 5.** The fidelity of the two-qubit gate in the presence of the $\Delta\omega$ fluctuation with various $\sigma_0$ when the input state is $(\cos\frac{\theta}{2}|+\rangle + \sin\frac{\theta}{2}|-\rangle)_{\mathrm{C}} \otimes |0\rangle_{\mathrm{T}}$. Parameters are: $\omega_{10}^a/2\pi \neq \omega_{10}^b/2\pi = 6$ GHz, $\Delta\omega/2\pi = \nu/2\pi = 300$ MHz, and $J/2\pi = 150$ MHz.

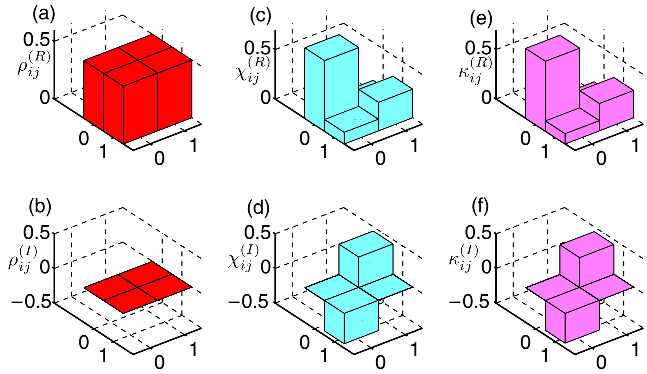


**Figure 6.** A graphical representation of the density matrices $\rho$, $\chi$, and $\kappa$ for the initial state, final state under the operation of ideal gate, and final state under the operation of a noisy gate, respectively, when the state $|1\rangle$ was measured in superconducting phase qubits. (a), (c), (e) and (b), (d), (f) denote, respectively, the real and imaginary parts, where $\theta = \pi/2$, $\gamma_{\mathrm{g}}^0 = \pi/8$, and $\sigma_0 = 0.1$. The fidelity of the noisy adiabatic gate is about 0.986.

target qubit is derived as

$$\gamma_{\mathrm{g}}^{+} = \frac{1}{2}\int_{\frac{\pi}{2}}^{\frac{5\pi}{2}} \frac{\nu^2 - \frac{1}{2}a\sin\phi}{(\sqrt{-a\sin\phi + b})(\sqrt{-a\sin\phi + b} + \Delta\omega)}\,\mathrm{d}\phi,$$
$$\gamma_{\mathrm{g}}^{-} = \frac{1}{2}\int_{\frac{\pi}{2}}^{\frac{5\pi}{2}} \frac{\nu^2 + \frac{1}{2}a\sin\phi}{(\sqrt{a\sin\phi + b})(\sqrt{a\sin\phi + b} + \Delta\omega)}\,\mathrm{d}\phi, \tag{7}$$

where $a = \nu J, b = \nu^2 + \Delta\omega^2 + \frac{1}{4}J^2$. Although $\gamma_{\mathrm{g}}^{+} = \gamma_{\mathrm{g}}^{-} = \gamma/2$, we can still have a nontrivial two-qubit controlled geometric phase gate, given by

$$U_{\mathrm{ctrl}} = \begin{pmatrix} U_{(\gamma,\xi^+)} & 0 \\ 0 & U_{(\gamma,\xi^-)} \end{pmatrix}, \tag{8}$$

where $\xi^{\pm} = \tan^{-1}[(\nu \mp J/2)/\Delta\omega]$.

We now numerically simulate the fidelity of the two-qubit gate by assuming only $\Delta\omega$ fluctuations, which are believed to be more significant than $\nu$ fluctuations in superconducting phase qubits. For brevity but without loss of generality, the input state is assumed to be $(\cos\frac{\theta}{2}|+\rangle + \sin\frac{\theta}{2}|-\rangle)_{\mathrm{C}} \otimes |0\rangle_{\mathrm{T}}$, with subscripts C and T denoting the states of the control and target qubits, respectively. Indeed, for the typical experiment parameters: $\omega_{10}/2\pi \sim 6$ GHz, $\Delta\omega/2\pi = \nu/2\pi = 300$ MHz, $C_x \sim 33$ fF, $C_{\mathrm{J}} \sim 1.3$ pF in [8, 27, 33], we have $J/2\pi \sim 150$ MHz. The fidelity of the two-qubit gate in the presence of $\Delta\omega$ fluctuation with various $\sigma_0$ is shown in figure 5. The calculated fidelity is also high (larger than 0.972) under the simulated noises. In this sense, we may state that the present geometric two-qubit gate is also robust against the random errors resulting from thermal fluctuation in superconducting phase qubits.

## 4. Detection of the Berry phase in superconducting phase qubits

Superconducting qubits, considered as artificial macroscopic two-level atoms, are also proposed as a candidate for detecting the geometric phase in macroscopic quantum systems [16, 17]. However, these existing proposals suggest detecting the Berry phase through the interference measurement, in which the dephasing may affect seriously the visibility in measuring this phase. In particular, it seems quite difficult to detect the Berry phase via the interference measurement in superconducting phase qubits. Recently, Steffen *et al* [27] reported the first demonstration of quantum state tomography using single shot measurements in superconducting phase qubits. Stimulated by this experiment, we here propose to directly detect the adiabatic Berry phase and to measure the fidelity of the designed geometric quantum gate via the quantum state tomography in future experiments.

Let us illustrate an example below. The initial state of a qubit is prepared as $|\psi_{\mathrm{i}}\rangle = [\cos(\theta/2), \sin(\theta/2)]^{\mathrm{T}}$ in the basis of $[|\psi_+\rangle, |\psi_-\rangle]$ (i.e. $[|0\rangle, |1\rangle]$ if we set $\nu(0) = 0$). As we described before, we drive the field to loop twice in a designated way in the parameter space, and thus the final state is $|\psi_{\mathrm{f}}\rangle = [\mathrm{e}^{2\mathrm{i}\gamma_{\mathrm{g}}^0}\cos(\theta/2), \mathrm{e}^{-2\mathrm{i}\gamma_{\mathrm{g}}^0}\sin(\theta/2)]^{\mathrm{T}}$. The first excitation $|\psi_-\rangle\langle\psi_-|$ was measured for reconstructing the density matrix of the state. The relative phase change between $|\psi_+\rangle$ and $|\psi_-\rangle$ observed through the state tomography is $4\gamma_{\mathrm{g}}^0$. As for superconducting phase qubits, the computational basis is $[|0\rangle, |1\rangle]$. The initial state of the qubit is prepared as $|\psi_{\mathrm{i}}\rangle = [\cos(\pi/4), \sin(\pi/4)]^{\mathrm{T}}$. After the two loops of adiabatic cone-type cyclic evolution, the final state $|\psi_{\mathrm{f}}^{\mathrm{ideal}}\rangle = [0.5 - \mathrm{i}(7 + \sqrt{15})/16, 0.5 + \mathrm{i}(7 - \sqrt{15})/16]^{\mathrm{T}}$ is obtained. Assuming the fluctuations of bias current are $\sigma_0 = 0.1$ and setting $\nu/\Delta\omega = \sqrt{15}/7$, from figure 4, the fidelity of the noisy adiabatic geometric gate is about 0.986 and the average obtained final state under the operation of the noisy adiabatic geometric gate is about $|\psi_{\mathrm{f}}^{noise}\rangle = [0.4872 - 0.6842\mathrm{i}, 0.4871 + 0.1968\mathrm{i}]^{\mathrm{T}}$. One rotates the Bloch vector of the qubit state in a Bloch sphere with microwave current pulses along $x$, $y$, and $z$ directions and measures the $|1\rangle\langle 1|$. The corresponding qubit state can be graphically represented [31], as shown in figure 6. From this figure, the Berry phase may be determined from the relative phase of the density matrix elements in (or between) the final state (and the initial state). Due to the influence of noise current, there are minor differences between the ideal final state $\chi$ and the final state $\kappa$ under the operation of a noisy gate. In experiments, one may optimize the experimental result by some methods which help to reduce the unavoidable decoherence and statistical errors [32]. In addition, the target qubit conditional phase shift may be detected by the simultaneous joint measurement of the two-qubit state [8, 33].

## 5. Discussions and summary

The experimental realization of our proposal for adiabatic gates and detecting the Berry phase in superconducting phase qubits is quite possible, although it may meet various technological challenges. The rapid inversion of the bias magnetic field to cancel the dynamical contribution to the overall phase is experimentally feasible. In fact, with a flux-biased superconducting phase qubit (which is essentially a current-biased Josephson junction) loop size of 50 $(\mu\mathrm{m})^2$ [8, 34], changing the flux by about half of a flux quantum in $10^{-10}$ s requires sweeping the magnetic field at a rate of about $2 \times 10^5$ T s$^{-1}$, which is reachable by current techniques [35]. If we can rapidly invert the bias magnetic field, the phase qubit remains in its instantaneous eigenstate.

Perhaps the main challenge is the implementation of the adiabatic evolution of the Hamiltonian to get the Berry phase within the qubit's decoherence time, which in turn must be longer than the typical timescale of superconducting phase qubits: $2\pi/\omega_{10}$, $2\pi/(\omega_{10} - \omega_{21}) \sim 3$ ns. The slowly varying phase of microwave current could be realized with 100 linear steps of about 4 ns. The required microwave technique is rather mature [8, 27]. In view of the decoherence time data in [5] and [27], it seems feasible to detect the geometric phase in phase qubits with the current quantum state tomography technology.

On the other hand, the thermal noise current in a superconducting qubits circuit is a white noise, homogeneous frequency broadening in the control current pulses. The thermal correlation time $\tau_{\mathrm{T}}$ is $\hbar/2\pi kT$ [36]. For the typical conditions for the superconducting qubits experiment, with the temperature of bias resistor $T \sim 4.2$ K, we have $\tau_{\mathrm{T}} \sim 2.89 \times 10^{-12}$ s. The ratio between the time required for adiabatic evolution $T_{\mathrm{ad}}$ and the thermal correlation time $\tau_{\mathrm{T}}$ is $T_{\mathrm{ad}}/\tau_{\mathrm{T}} \gg 1$. This means there are fast varying random fluctuations. Therefore, it is reasonable that the stochastic numbers are chosen to be more than 10 000 when we calculate the fidelity under numerical simulation of the noise [22].

As for a direct comparison of the adiabatic gates and the dynamic gates in superconducting qubits, to our knowledge, it was indicated previously that the geometric gates are more robust against fluctuations of control parameters than dynamic gates [23]. From our numerical simulations, adiabatic geometric gates are likely to be robust against the random errors caused by the weakly fluctuating driving bias current in superconducting phase qubits. We wish to indicate that although there are limitations caused by the adiabatic condition, geometric QC based on the adiabatic Berry phase may have an interesting application in a precise preparation of a quantum state [17, 23], mainly due to its global geometric robustness against certain kinds of errors. An experimental process to determine the noisy channel of the controlled qubits based on the qubit state tomography is referred to as quantum process tomography [1, 31]. Since a set of standard qubit states must be precisely prepared in the quantum process tomography experiments, we may use the present geometric QC strategy to achieve them. For example, to realize quantum process tomography for a single phase qubit, the four kinds of input states $|0\rangle$, $|1\rangle$, $|+\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$ and $|-\rangle = (|0\rangle + \mathrm{i}|1\rangle)/\sqrt{2}$ need to be precisely prepared. In our scheme, the state $|-\rangle$ can be made from an easy initial state $|0\rangle$ once we set $\varphi_i = 0$, $\theta_i = 0$, $\xi = \pi/2$, and $\gamma_{\mathrm{g}}^0 = \pi/8$, with a relatively high fidelity for weaker noises.

In summary, we have developed an adiabatic geometric QC strategy based on the non-degenerate energy eigenstates and have especially considered construction of two-qubit adiabatic geometric gate based on $\sigma_y\sigma_y$ coupling. The fidelity of the designed quantum gate has been evaluated in the presence of simulated Gaussian-type thermal fluctuation noises in superconducting phase qubits and has been found to be rather robust against random errors. A possible application of our strategic scheme in a precise preparation of a designated quantum state has been addressed. We have also proposed to detect directly the Berry phase in phase qubits via the quantum state tomography.

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*Note added.* After completion of this work, we learned that Wallraff *et al* reported observing the Berry phase in superconducting charge qubits with microwave techniques via quantum state tomography [37].

## References

[1] Nielsen M A and Chuang I L 2000 *Quantum Computation and Quantum Information* (Cambridge: Cambridge University Press)
[2] Makhlin Y, Schön G and Shnirman A 2001 *Rev. Mod. Phys.* **73** 357
[3] Nakamura Y, Pashkin Yu A and Tsai J S 1999 *Nature* **398** 786
[4] Chiorescu I, Nakamura Y, Harmans C J P and Mooij J E 2003 *Science* **299** 1869
[5] Yu Y, Han S, Chu X, Chu S and Wang Z 2002 *Science* **296** 889
[6] Martinis J M, Nam S, Aumentado J and Urbina C 2002 *Phys. Rev. Lett.* **89** 117901
[7] Yamamoto T, Pashkin Y A, Astafiev O, Nakamura Y and Tsai J S 2003 *Nature* **425** 941
[8] Steffen M, Ansmann M, Bialczak R C, Katz N, Lucero E, McDermott R, Neeley M, Weig E M, Cleland A N and Martinis J M 2006 *Science* **313** 1423

[9] Niskanen A O, Harrabi K, Yoshihara F, Nakamura Y, Lloyd S and Tsai J S 2007 *Science* **316** 723

[10] Plantenberg J H, de Groot P C, Harmans C J P M and Mooij J E 2007 *Nature* **447** 836

[11] Sillanpää M A, Park J I and Simmonds R W 2007 *Nature* **449** 438

[12] Steane A 2002 arXiv:quant-ph/0207119

[13] Leibfried D, DeMarco B, Meyer V, Lucas D, Barrett M, Britton J, Itano W M, Jelenkovic B, Langer C, Rosenband T and Wineland D J 2003 *Nature* **422** 412

[14] Du J F, Zou P and Wang Z D 2006 *Phys. Rev.* A **74** 020302

[15] Berry M V 1984 *Proc. R. Soc.* A **392** 45

[16] Falci G, Fazio R, Palma G M, Siewert J and Vedral V 2000 *Nature* **407** 355

[17] Peng Z H, Zhang M J and Zheng D N 2006 *Phys. Rev.* B **73** 020502(R)

[18] Zhang P, Wang Z D, Sun J D and Sun C P 2005 *Phys. Rev.* A **71** 042301

[19] Zhu S L and Wang Z D 2002 *Phys. Rev. Lett.* **89** 097902
Zhu S L and Wang Z D 2002 *Phys. Rev.* A **66** 042322

[20] Zanardi P and Rasetti M 1999 *Phys. Lett.* A **264** 94
Pachos J, Zanardi P and Rasetti M 2000 *Phys. Rev.* A **61** 010305

[21] Zhang X D, Zhu S L, Hu L and Wang Z D 2005 *Phys. Rev.* A **71** 014302
Shao L B, Wang Z D and Xing D Y 2007 *Phys. Rev.* A **75** 014301

[22] Solinas P, Zanardi P and Zanghi N 2004 *Phys. Rev.* A **70** 042316

[23] Zhu S and Zanardi P 2005 *Phys. Rev.* A **72** 020301(R)

[24] Clarke J, Cleland A N, Devoret M H, Esteve D and Martinis J M 1988 *Science* **239** 992

[25] Bertet P, Chiorescu I, Burkard G, Semba K, Harmans C J P M, DiVincenzo D P and Mooij J E 2005 *Phys. Rev. Lett.* **95** 257002

[26] Horowitz P and Hill W 1989 *The Art of Electronics* (Cambridge: Cambridge University Press)

[27] Steffen M, Ansmann M, McDermott R, Katz N, Bialczak R C, Lucero E, Neeley M, Weig E M, Cleland A N and Martinis J M 2006 *Phys. Rev. Lett.* **97** 050502

[28] Martinis J M, Nam S, Aumentado J, Lang K M and Urbina C 2003 *Phys. Rev.* B **67** 094510

[29] Jones J A, Vedral V, Ekert A and Castagnoli G 2000 *Nature* **403** 869

[30] Pachos J K and Beige A 2004 *Phys. Rev.* A **69** 033817

[31] Liu Y, Wei L F and Nori F 2005 *Phys. Rev.* B **72** 014547

[32] Paris M G A and Řeháček J 2004 *Quantum State Estimation* (Berlin: Springer)

[33] McDermott R, Simmonds R W, Steffen M, Cooper K B, Cicak K, Osborn K D, Oh S, Pappas D P and Martinis J M 2005 *Science* **307** 1299

[34] Simmonds R W, Lang K M, Hite D A, Nam S, Pappas D P and Martinis J M 2004 *Phys. Rev. Lett.* **93** 077003

[35] Uwazumi H, Shimatsu T and Kuboki Y 2002 *J. Appl. Phys.* **91** 7095

[36] Gardiner C W and Zoller P 2000 *Quantum Noise* (Berlin: Springer)

[37] Leek P J, Fink J M, Blais A, Bianchetti R, Göppl M, Gambetta J M, Schuster D I, Frunzio L, Schoelkopf R J and Wallraff A 2007 *Science* **318** 1889